Gravity and Orbital Physics

Dan Burns
Los Gatos High School
dburns@lgsuhsd.org

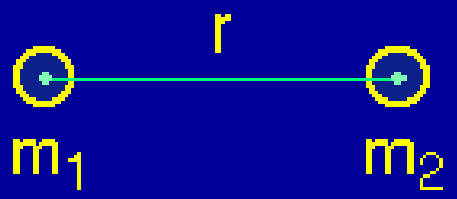
Agenda

- Newton's Law of Gravity
- Free Fall
- Circular Motion
- Circular Orbits
- Kepler's Laws
- General Relativity

Law of Universal Gravitation

Every object in the Universe attracts every other object with a force directed along the line of centers for the two objects that is proportional to the product of their masses and inversely proportional to the square of the separation between the two objects.

$$F_g = G\frac{m_1 m_2}{r^2}$$



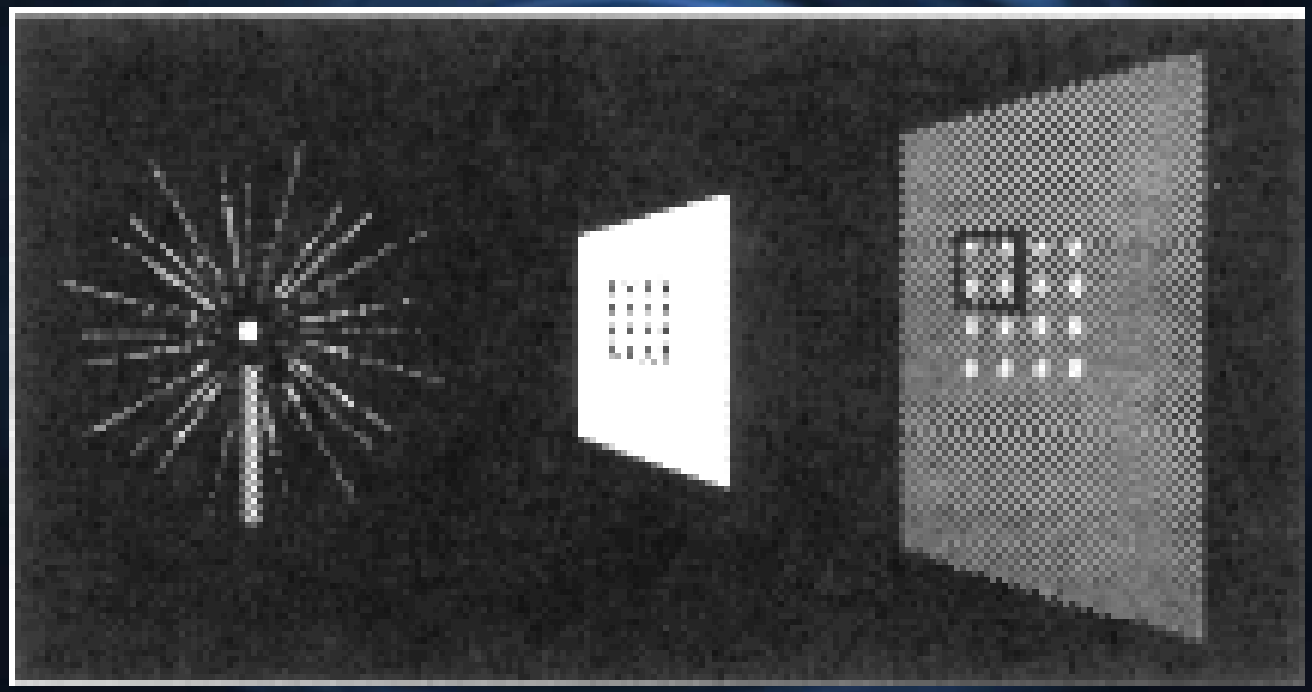
F_g is the gravitational force

m_1 & m_2 are the masses of the two objects

r is the separation between the objects

G is the universal gravitational constant

Perfboard Inverse Square Law Activity

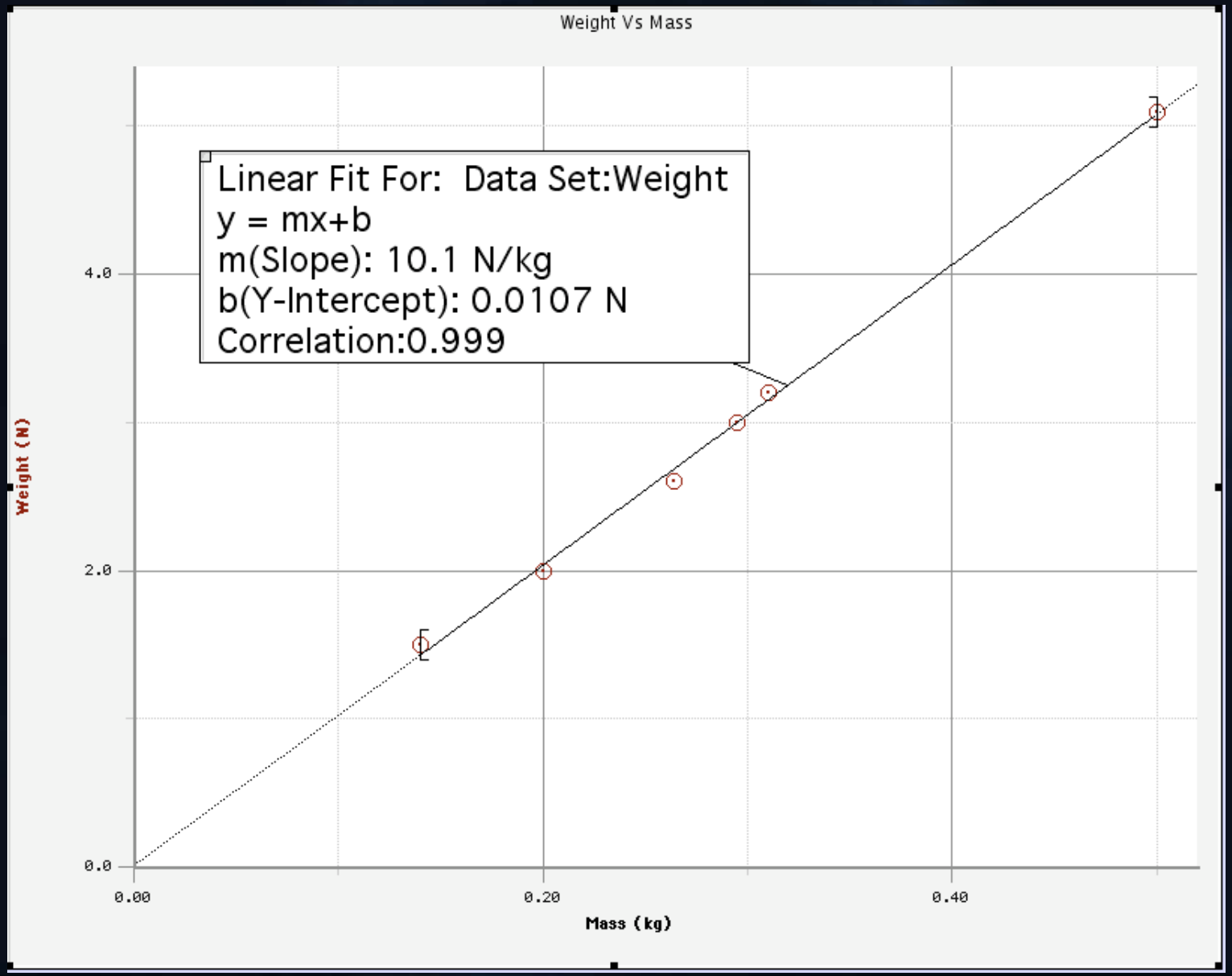


http://www.exploratorium.edu/snacks/inverse_square_law/index.html

Measuring g, the Gravitational Field Strength on Earth

- Provide students with a variety of objects from about 0.1 to 2.0 kg, a spring scale, and a triple-beam balance
- Have students measure weight in Newtons with scale and mass in kilograms with balance
- Graph weight on y-axis and mass on x
- Measure slope of best-fit line, should be about 9.8 N/kg

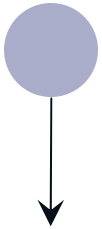
Sample Data for Weight Vs Mass



A ball is thrown straight up. What is its acceleration at the highest point ?

A.Zero
B.Very small, but not zero
C.9.8 m/s^2 down
D.9.8 m/s^2 up
E.Depends on air resistance

Free-fall Acceleration



$$\sum F = ma$$

$$mg = ma$$

$$a = g$$

- All Objects Fall with the Same Acceleration
- This Acceleration (g) is 9.8 m/s^2 on the Earth's Surface
- This Acceleration is Constant in Magnitude and Direction

What is the Relationship between Distance and Time When Dropped?

$$\bar{V} = \frac{V_0 + V}{2}$$

$$V_0 = 0$$

$$V = at$$

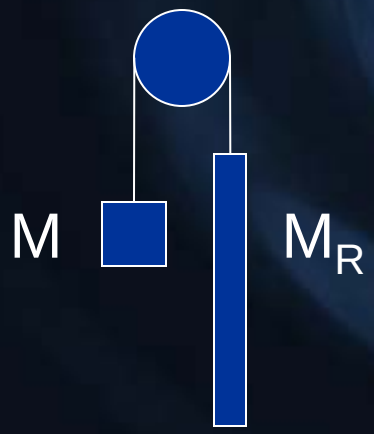
$$D = \bar{V}t = \frac{at}{2}t = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2D}{a}}$$

- Average Velocity is the Average of the Initial and Final
- The Final Velocity is the Acceleration Multiplied by Time
- The Distance is the Average Velocity Multiplied by Time

Free Fall Activities

- Reaction Time Measurement
- Repeat with added Weight
- Repeat on the Moon!



$$M = \frac{M_R(9.8 - g_{moon})}{(9.8 + g_{moon})}$$

$$= \frac{.156(9.8 - 1.7)}{(9.8 + 1.7)} = .11kg$$

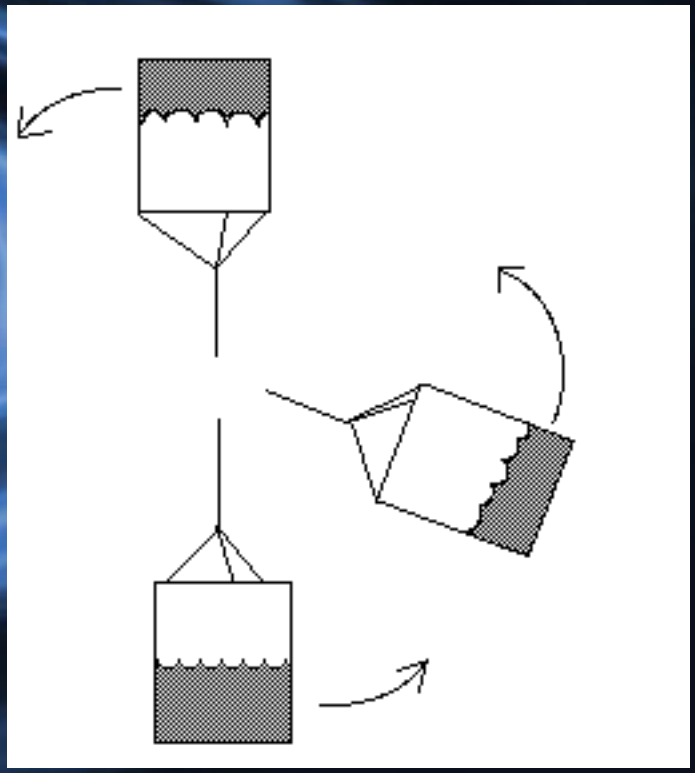
Circular Motion

- Force = Mass x Acceleration
- Acceleration is rate of change of velocity, (speed and/or direction)
- Net Force and Acceleration are always in the same direction (demo)
- This is toward the center (centripetal) for uniform circular motion

$$a_c = \frac{v^2}{r}$$

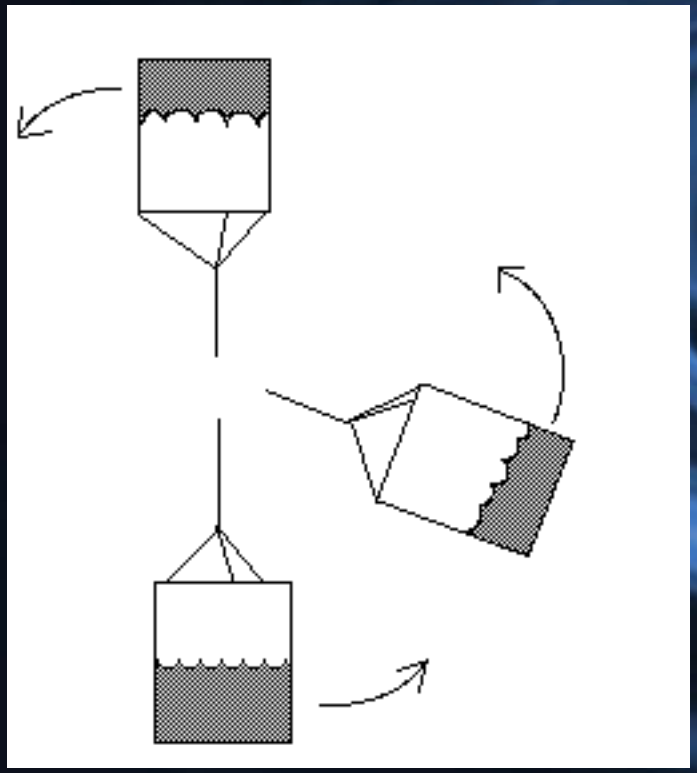
A bucket of water is spun in a vertical circle. What keeps the water from falling?

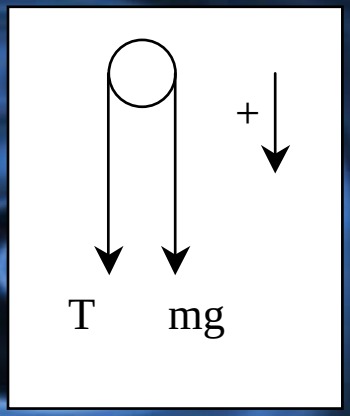
A.Centrifugal force
B.Centripetal force
C.The Normal force
D.Gravity
E.None of the Above



Spinning Bucket of Water

How Fast to Keep Water from Coming Out?





T = 0 at Minimum Speed

$$\sum F = ma_c$$

$$T + mg = m\frac{v^2}{r}$$

$$\frac{v^2}{r} = g$$

$$v = \sqrt{gr}$$

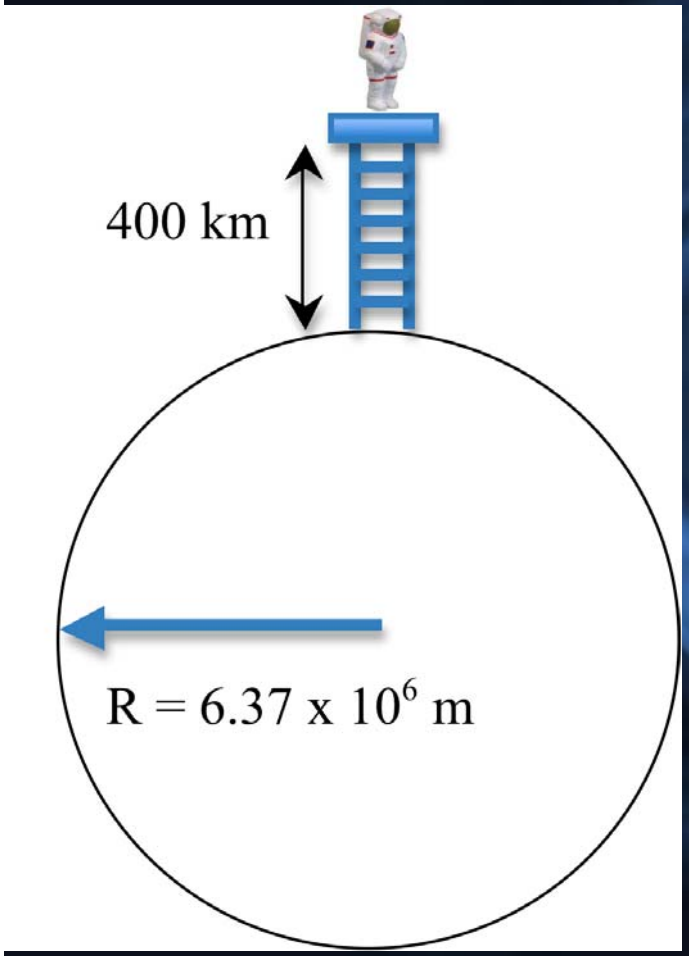
An astronaut lets go of a screwdriver while orbiting the Earth. It floats before her eyes. The same thing can be done

A.In an airplane
B.On an amusement park ride
C.In a high school classroom
D.All of the above
E.None of the above

Apparent Weightlessness

- F_g on Earth's surface and 400 km altitude
- How do we get bathroom scale to read zero?
- Free Fall Demos
- Gravity DVD
- Gravity Switch

Bathroom Scale on a 400 km Ladder



$$F_g(\text{on surface}) = \frac{GMm}{r^2} =$$

$$\frac{6.67\times10^{-11}\times5.98\times10^{24}\times90}{(6.36\times10^{6})^2} = 885N$$

$$F_g(\text{on ladder}) = \frac{GMm}{r^2} =$$

$$\frac{6.67\times10^{-11}\times5.98\times10^{24}\times90}{(6.36\times10^{6}+4\times10^{5})^2} = 783N$$

$$= 88\%\ \textit{surface value}!$$

Free Fall Demos

Gravity Video Clip DVD

- Vomit Comet Clip
- Overview of Contents
- How to Get One:

Send Stamped, Self-Addressed DVD Mailer and One Blank DVD, any format, To:

Dan Burns
Los Gatos High School
20 High School Court
Los Gatos, CA 95030

Gravity Switch

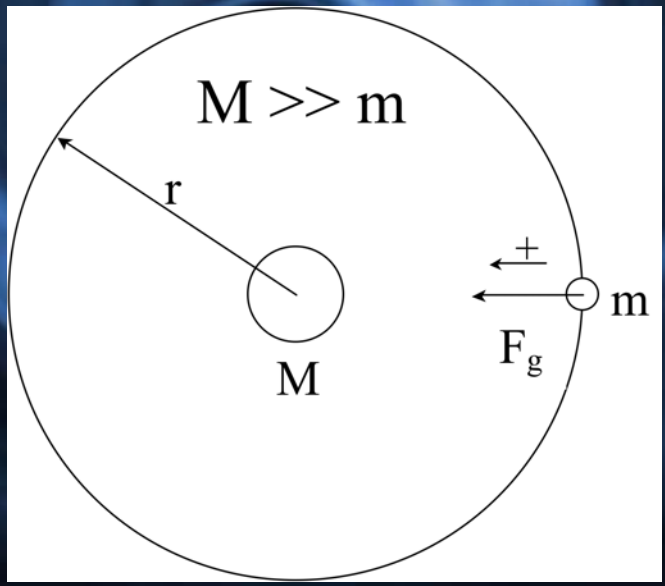
- How to Make One
- What Would Happen if it is Turned Off?
- How to Simulate Result

Circular Orbit Speed and Period Derivation

$$\sum F = ma_c$$

$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$



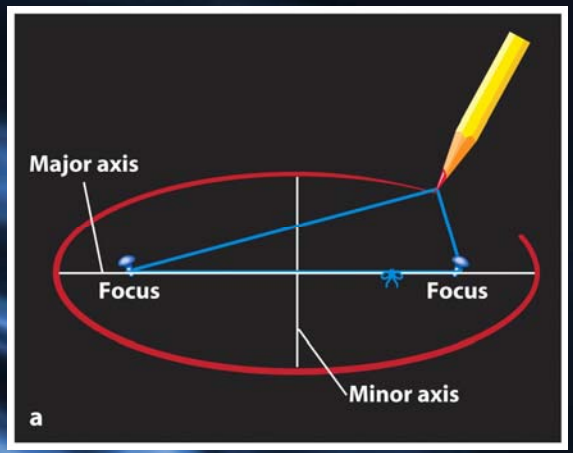
$$v = \frac{\Delta s}{\Delta t}$$

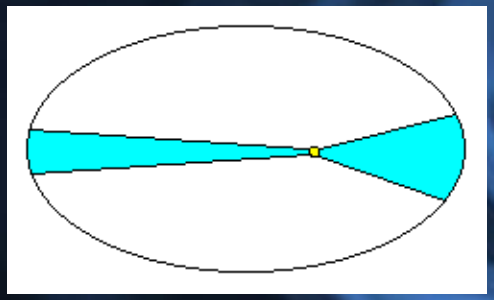
$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{\mathrm{T}}$$

$$\mathrm{T} = 2\pi\sqrt{\frac{r^3}{GM}}$$

Kepler's Laws

1. Orbits are Ellipses





2. Equal Areas are Swept out in Equal Times

3. The square of the Period is proportional to the cube of the average distance

$$T^2 = \frac{4\pi^2}{GM} r^3$$

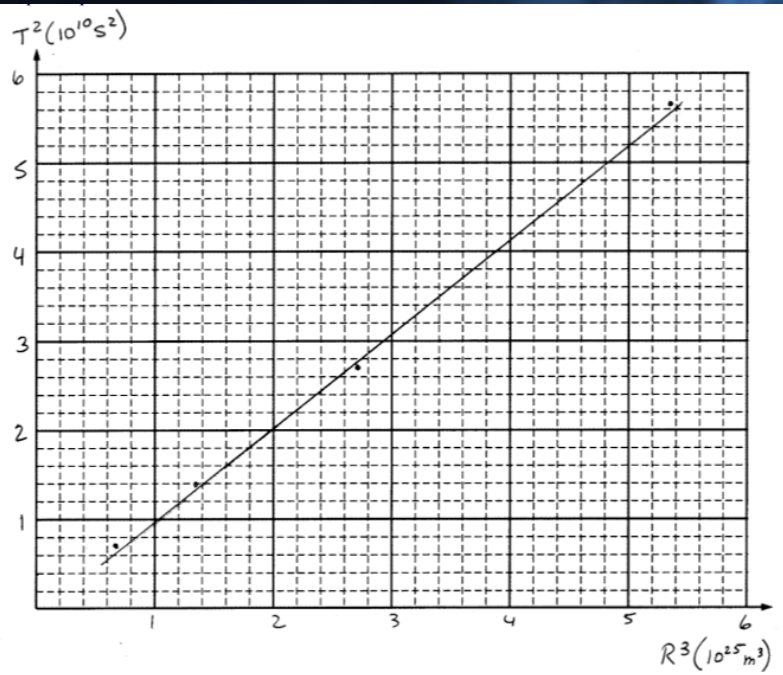
Discovering Kepler's 3rd Law and the Mass of a Planet

- Obtain period and radius data of moons
- Graph period Vs radius
- Modify data until a straight line results
- Determine slope of line
- Using Kepler's 3rd Law and y = mx+b, calculate mass of Planet

Data for Four of Saturn's Moons

Example data

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 (s^2)	R^3 (m^3)
8.14×10^4	1.85×10^8	0.663×10^{10}	0.633×10^{25}
1.18×10^5	2.38×10^8	1.39×10^{10}	1.35×10^{25}
1.63×10^5	2.95×10^8	2.66×10^{10}	2.57×10^{25}
2.37×10^5	3.77×10^8	5.62×10^{10}	5.36×10^{25}



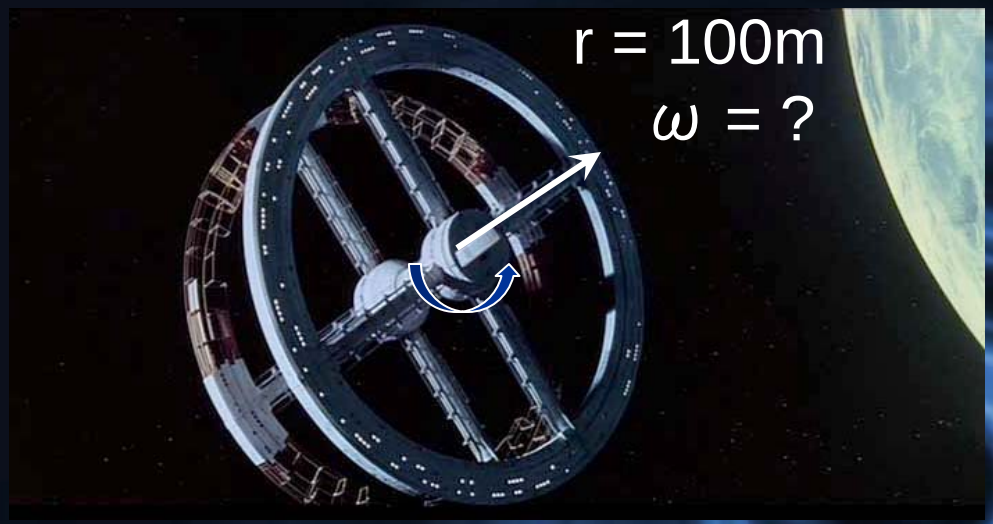
$$M_S = \frac{4\pi^2}{G\ (\text{slope})}$$

$$\text{slope} = \frac{\left(5.6 \times 10^{10} - 1.4 \times 10^{10}\right)\text{m}^3}{\left(5.4 \times 10^{25} - 1.4 \times 10^{25}\right)\text{s}^2} = \frac{4.2 \times 10^{10}\ \text{m}^3}{4.0 \times 10^{25}\ \text{s}^2}$$

$$M_S = \frac{4\pi^2}{6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}} \frac{4.0 \times 10^{25}\ \text{s}^2}{4.2 \times 10^{10}\ \text{m}^3}$$

$$M_S = 5.64 \times 10^{26}\ \text{kg}$$

Artificial Gravity



$$\sum F = ma_c$$

$$N = m\omega^2 r$$

For Earth Gravity:

$$N = mg$$

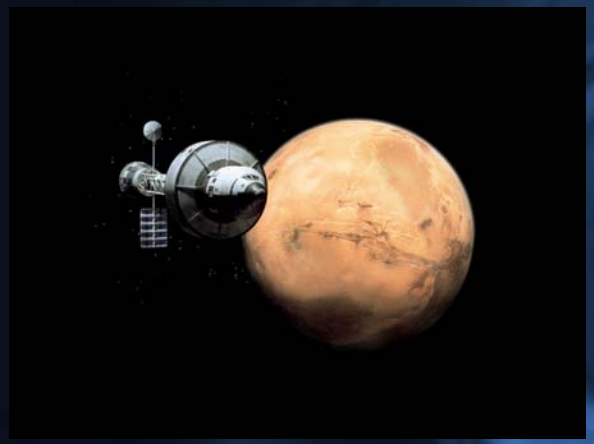
$$\omega = \sqrt{g/r}$$

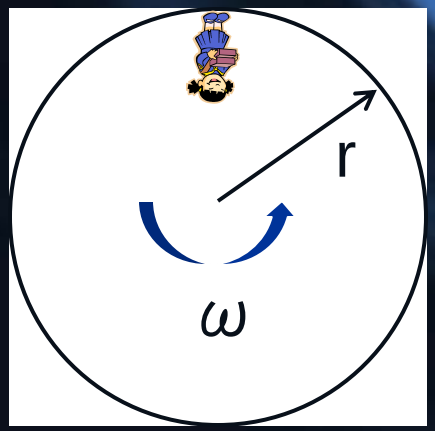
$$\omega = \sqrt{9.8/100} =$$

$$0.31\ r/s = 3\ rpm$$

Mission to Mars

What is artificial gravity level on Mission to Mars Movie spacecraft?





Height of Astronaut = 1.8m
Radius = 2.5 Astronauts = 4.5m
Time for 3/8 of a Rotation = 10.9s
Time for 1 Rotation (T) = 29s
$\omega = 2\pi / T = 0.22$ rad/s
Acceleration (Feet) = $\omega^2 r = 0.22$ m/s^2
Acceleration (Head) = $\omega^2 r = 0.13$ m/s^2

Acceleration is too small to help astronauts, difference could produce nausea

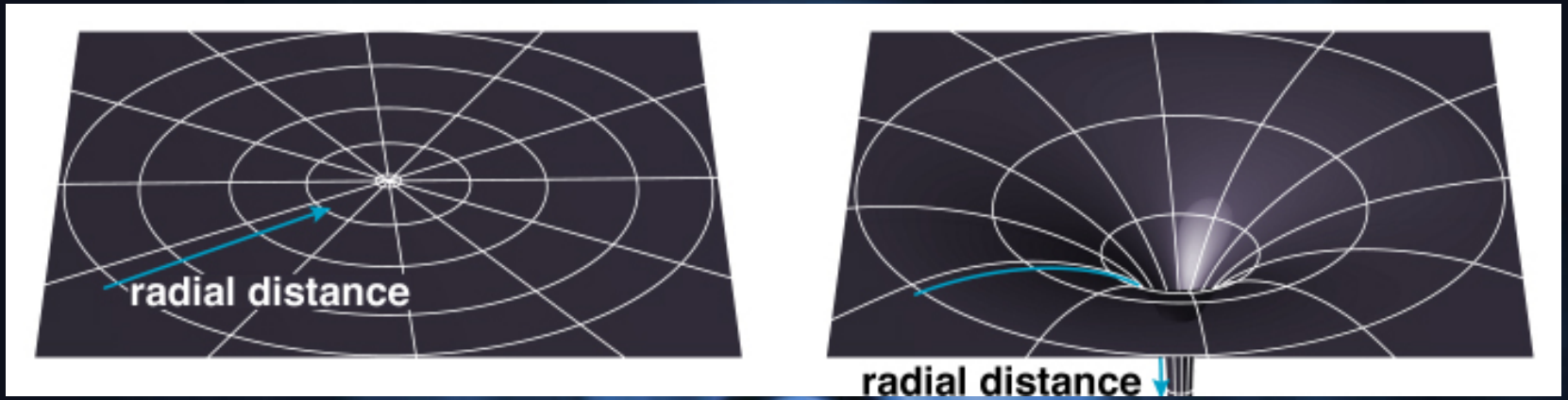
Online Gravity Simulation

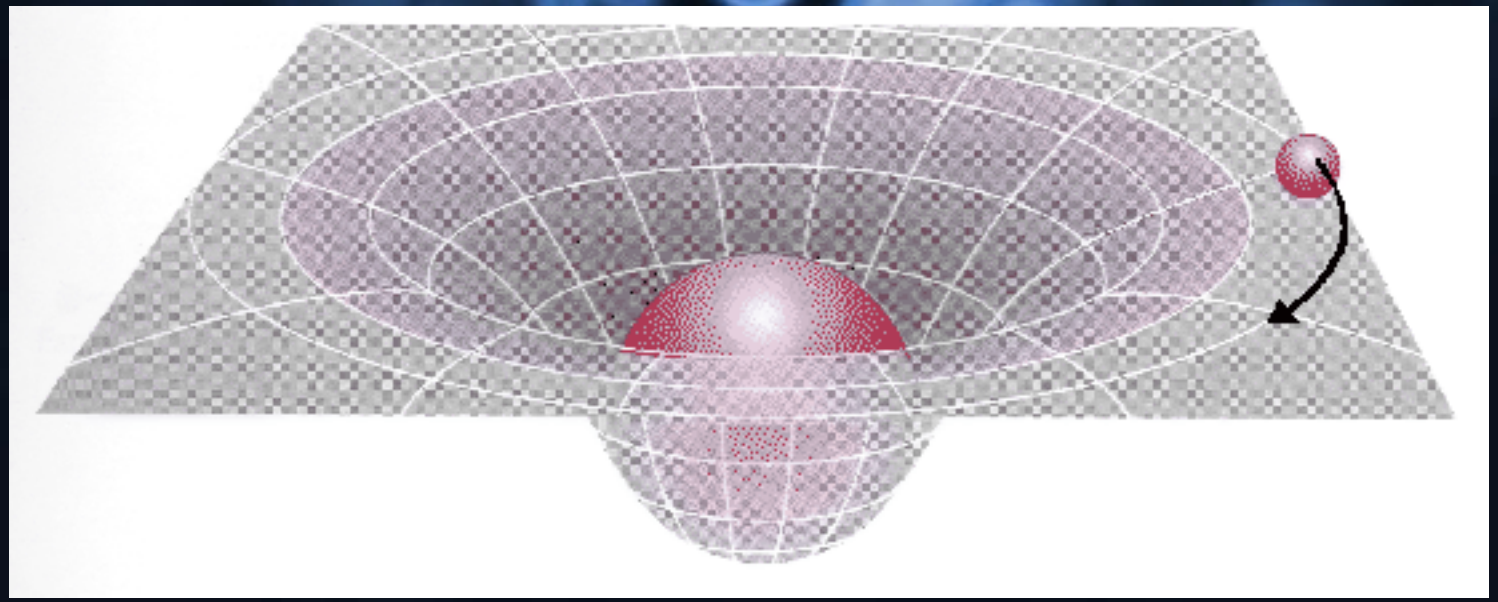
- My Solar System on the University of Colorado Website
- Simulates gravity between up to 4 objects
- Pre-loaded simulations and user created
- Downloadable teacher developed activities, including one by D. Burns
- http://phet.colorado.edu/simulations/sims.php?sim=My_Solar_System

General Relativity

- Theory, not a law, of gravity
- Mass curves space-time
- Mass and light follows curvature
- Curvature masquerades as force of gravity
- Experimental tests of General Relativity
- Curved Space-time simulator

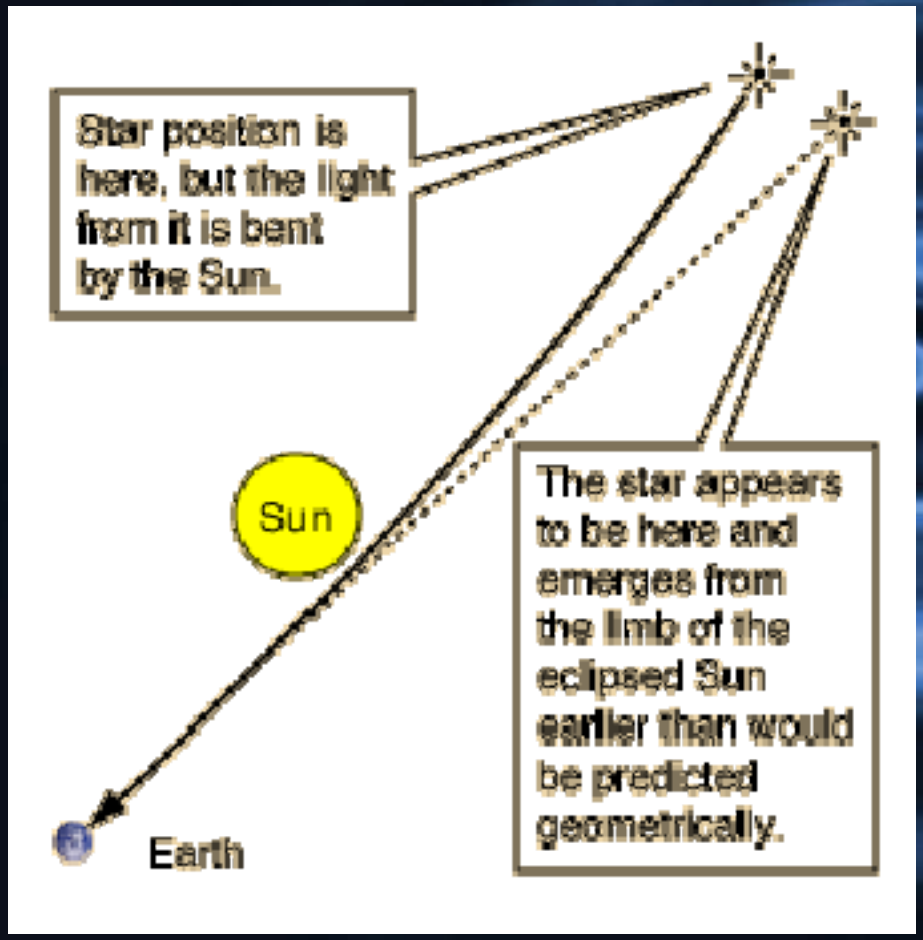
Curvature of Space-Time





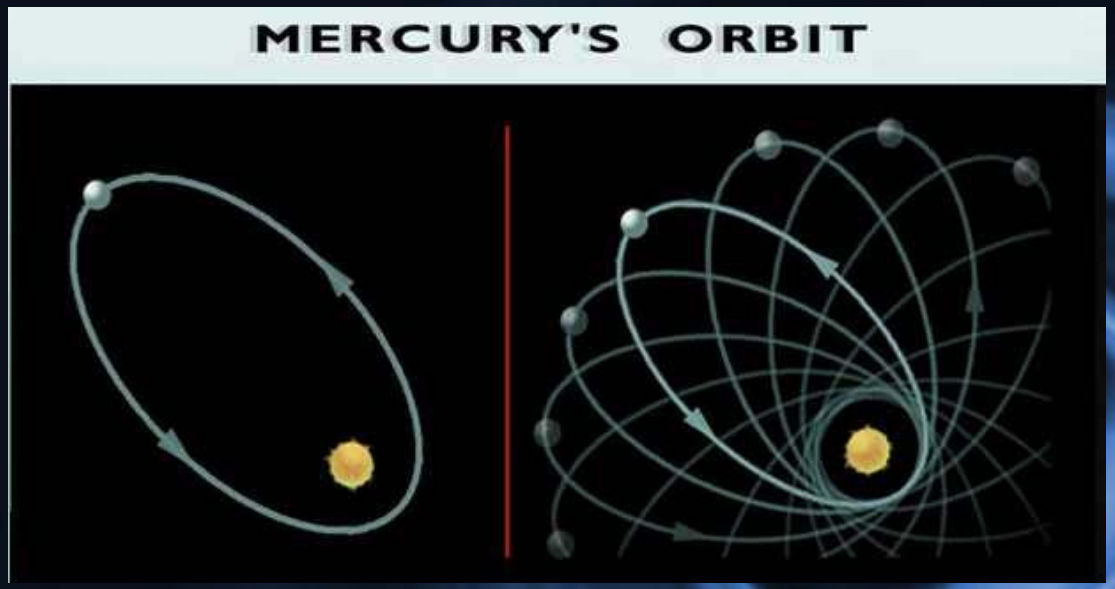
http://psi.phys.wits.ac.za/teaching/Connell/phys284/2005/lecture-01/lecture_01/node17.html

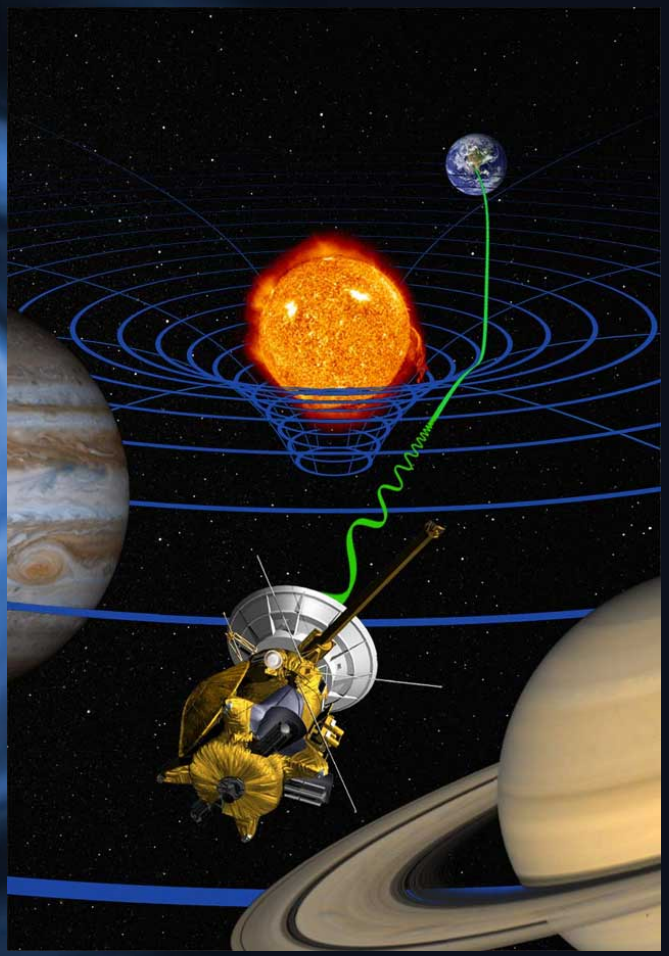
Gravity Bends Light



http://archive.ncsa.uiuc.edu/Cyberia/NumRel/Movies/MCNC_blackhole1.mov

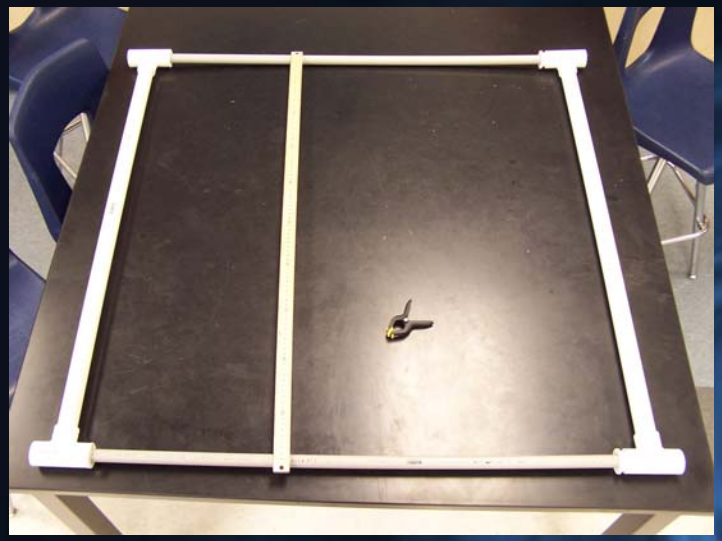
General Relativity Tests





http://saturn.jpl.nasa.gov/news/press-releases-03/images/cassini-science-br.jpg

Curved Space-time Simulator



7 36” 1.25” PVC Pipes

14 PVC T-Connectors

14 34” .75” PVC Electrical Conduit

28 .75” to 1.25 “ Connector (PVC-1 D2466 IPS 1 x 1/2)

~ 70” x 70” piece of Lycra (your old bike shorts!)